PIEZOELECTRIC HIGH VOLTAGE TRANSFORMERS

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1. INTRODUCTION

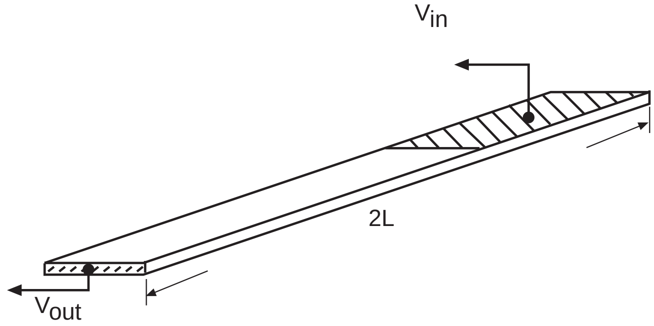
It appears an appropriate time to consider some of the formulas pertaining to the performance of piezoelectric transformers and their relation to high amplitude material characteristics. Factors which could conceivably limit performance of piezoelectric transformers are listed below.

1. Internal mechanical losses due to finite mechanical quality (Q_M).
2. Internal dielectric losses due to finite electrical quality ($Q_E = 1/\tan\partial$)
3. Mechanical breakage.
4. Dielectric breakdown.
5. Frequency drift due to internal heating.

Items 1) and 3) are important, since the device must operate at high dynamic stress levels. Item 4) is of little consequence, but we shall see that electric field levels are high enough in the output section that item 2) is important. Item 5) is, from a practical standpoint, also important. If the horizontal scan frequency is used, probably an economic necessity, a change of resonance frequency due to heating will cause fluctuation in the output voltage. Some fluctuation can certainly be tolerated because the eye can accommodate to changes in brightness. If a separate oscillator controlled by the ceramic element is used, drift is not a problem.

2. ANALYSIS WITHOUT ELECTRIC LOAD
(Open-Circuit Secondary)

Using the configuration shown below, simplified (and approximate) design formulas are as follows. In all cases peak values of stress, voltage, and electric field are used — not rms. This holds throughout this report.



(1) $(V_{out}/V_{in})_{open\text{-}ckt.} \approx 0.5 k_{31}\, k_{33}\, Q_M\, L/t$ and

(2) $P_{DM} \approx (0.5\, T_{ave}^2/Y)\; 2\pi f/Q_M$

where
PDM ≈ dissipated power density due to mechanical losses, watts/m
T_{ave} = space average of the mechanical stress (N/m^2)
Y = Youngs modulus (N/m^2)
g_{33} = piezoelectric constant (10^{-3} V m /N),
f = frequency in Hertz, and
Q_M = mechanical Q .

These formulas are inexact, primarily because they fail to take into account the essentially sinusoidal stress distribution and the differences in effective elastic wave velocities in the two sections of the transformer. An exactsolution would be extremely cumbersome, but it should be noted that effectively the driving section involves $1/S^E_{11}$ and its effective loss tangent whereas the output section involves $1/S^D_{33}$ and its effective loss tangent. For purposes of this discussion the difference in loss tangents is ignored. Y may be assumed to be the average of $1/S^E_{11}$ and $1/S^D_{33}$ and only one effective loss tangent, equal to $1/Q_M$, will be considered.*

* Our usual measurement scheme leads to
$Qm = S^E_{11} \,/\, S^D_{33}$
where $S^E_{11} = S^E_{11}{}' - jsS^E_{11}{}''$

In Eqs. (1) and (2) it must be recognized that Q_M, Y, d_{31} and k_{33} are all functions of stress T . The most important functional dependence is that of Q_M, which may decrease to less than one-fourth its low amplitude value. For purposes of analysis, therefore, it is sensible to neglect all the functional relationships but that of Q_M to stress.

The output voltage (open-circuit) is very simply related to dynamic stress, with stress related to input voltage approximately as listed below:

(3a) $V_{out} \approx g_{33} L T_{ave}$ and

(3b) $T_{ave} \approx 0.5\, E_{in}\, d_{31} Y Q_M$

where, as before, T_{ave} is the space average mechanical stress.* Substitution of (3) into (2) leads to the following relationship:

$$P_{DM} = \left(\frac{V_{out}^2}{g_{33}^2 L^2 Y}\right) \frac{\pi f}{Q_M}$$

Noting that frequency and length are inversely proportional,

(5) $f = N/2L$,

the following result is obtained:

$$P_{DM} = \left(\frac{V_{out}^2}{g_{33}^2 L^2 Y}\right) \frac{4\pi f^3}{Q_M}$$

This relationship shows clearly that the dissipated power- density. is proportional to the third power of frequency. For a given voltage, the mechanically dissipated power density at 31,500 Hz is therefore eight times that required at 15,750 Hz (assuming no change in Q_m).

Combination of Eqs. (3) and (5) gives

(7) $T_{ave} = \dfrac{2f\, V_{out}}{gN}$

Thus for a given output voltage, dynamic stress is directly proportional to frequency. Since Q_m decreases with stress, the dissipated power density of Eq. (6) actually increases even faster than f . Figure I shows Q_M VS. peak dynamic stress for PZT-8 rings manufactured by Piezoelectric Division. Two curves are shown, one obtained at room temperature, the other at 71°C (165°F). Temperature was maintained during the runs by using resonant drive. With PZT-4 Q_M values are typically about one-half those for PZT 8. Low stress values of Q_M in Fig. I are unusually low for PZT-8, but for stress greater than about 1000 psi, values are typical. The measurements of Fig. I were made with ceramic rings, which are uniformly stressed at resonance. The transformer sketched before is a bar with approximately sinusoidal stress distribution at resonance; in this case $T_{ave} = 2/\pi\, T_{centre}$*

It is important to note that the electric field in the output section is considerably higher than that in the input section. Combining Eqs. (l) and (3a), the ratio is found to be:

(8) $E_{output\,(open\text{-}ckt)} / E_{input} \approx 0.5 k_{31} k_{33} Q_M$.

With $Q_M = 500$ the ratio is thus found to be about 46 for PZT-8.

An example which can be used to predict actual performance will now be discussed briefly. Actual parameters involved are listed below.
(PZT-8)

k_{31}	=	-0.295
k_{33}	=	0.62
N	≈	1850 cycle m/sec
d_{31}	=	- 93 x.10^{-12} m/V
$\varepsilon T_{33}/\varepsilon_o$	=	1000
g_{33}	=	24.6 x 10^{-3} Vm/N
Q_M	=	f(T), see Fig. 1

$Y_{eff} = 10.4 \times 10^{10}$ N/m ($S^E_{11} = 11.1$, $S^D_{33} = 8.5 \times 10^{-12} m^2/N$)
$\rho = 7.5 \times 10^3\ kg/m^3$

For these purposes
$Y_{eff} = \frac{1}{2}\left(\frac{1}{S^E_{11}} + \frac{1}{S^D_{33}}\right)$ and $N = 0.5\sqrt{(Y_{eff}/\rho)}$

First assume that an open-circuit voltage of 5000 volts is required at 31.5 kHz. This sets L ≈ 3 cm. (Eq. 5) and $T_{ave} \approx 6.9 \times 10$ N/m, or 1000 psi (Eq. 7). Figure 1 is not directly applicable, since it shows Q_M with a uniform stress, while we have in this case an average stress of 1000 psi with a maximum of ≈ 1570 psi at the centre and zero at the ends.

It appears from Fig. 1 that Q_M should be about 500. The output electric field is 5/3 = 1. 67 kV/cm. The required input field is thus 27 volts/cm (Eq. 8). The mechanically dissipated power (Eq. 2) is 0.101 watts/cm^3, a respectably low figure.

At this point the dielectric counterpart to Eq. (1) should be considered in order to evaluate the effects of dielectric dissipation. We have seen that although mechanical stress is sinusoidally distributed throughout the bar, the average electric field in the output end is much higher than that at the input end. It is necessary only to consider the sinusoidally distributed field in the output end.

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*$T_{ave} \approx (2/\pi)T_{max}$, with T_{max} at the centre of the bar

* This relationship is disturbed somewhat by the different elastic properties of the two sections.

(9) $P_{DE} = 0.5E_{ave}\, \varepsilon T_{33} \tan \partial\, (2\pi f)$, *

where

E_{ave} = space average of the electric field in the output section; $V_{out} = E_{ave} L$

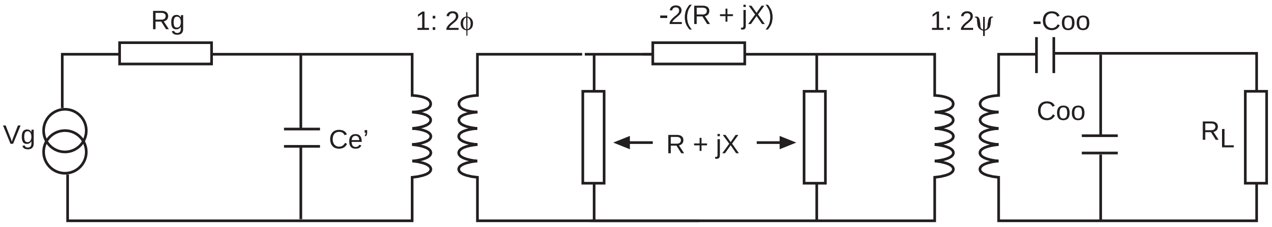
Since the electric field is not uniform in the output section, the previous discussion with respect to application of Eq. (1) for the stress again applies.

It is safe to say that $\tan\partial \approx 0.01$ for PZT-8 at this level of electric field. For the case at hand, Eq. (9) gives $P_{DE} = 0.271$ watts/cm^3. It should be noted that this loss occurs only in the output section**

Reduction of the frequency from 31.5 to 15.7 kHz would thus reduce P_{DM} and P_{DE} to approximately 0.01 and 0.03 watts/cm^3 respectively and would reduce the average stress and electric field to 500 psi and 0. 83 kV/cm.

3. ANALYSIS WITH ELECTRIC LOAD; ZERO IMPEDANCE GENERATOR DRIVE

The simplified equivalent circuit for the transformer is shown below.



The following list of parameters holds at resonance.

$C_{oi} = \varepsilon T_{33}(1-k_{31}^2)\, lw/t$ (Clamped capacitance of input section).

$C_{oo} = \varepsilon T_{33}(1-k_{33}^2)\, lw/t$ (Clamped capacitance of output section).

$\phi = wd_{31} / s^E_{11}$

$\psi = k_{33}^2\, wt/g_{33}L$

$R = (\pi/Q_M)\ /\rho vwt$

$X = 2\pi r\rho vwt$

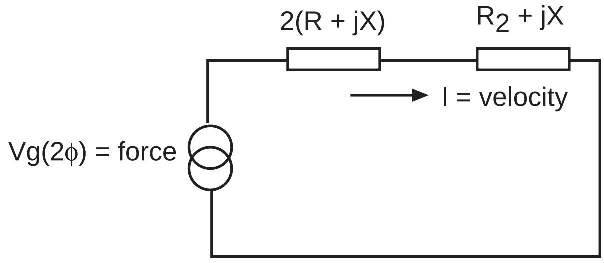
r = density, kg/m^3
v = velocity = 2N
r = $\Delta f/f$ (frequency deviation from resonance)

* Note here that for a given voltage, P_{DE} is proportional to f^3 just as is P_{DM} since E is proportional to f for a given voltage.

** Assuming no heat transfer or radiation.. PZT-8 may be expected to suffer a temperature rise of about 0.3 °C/watt second.

Dielectric losses may be considered part of the load resistance R_L and taken account of later.

Considerable simplification results if R_g is assumed to be zero. For this case the Thevenin equivalent referred to the mechanical section of the equivalent circuit is as follows:



Here

$$R_2 = \frac{R_L}{1 + R_L^2\, \omega^2\, c_{oo}^2}\ (4\psi^2)$$

$$X_2 = \frac{1}{\omega C_{oo}} / (1 + R_L^2\, \omega^2\, C_{oo}^2)\, 4\psi^2$$

R, X as before

Unfortunately the equivalent circuits shown here are far from accurate since they fail to take into account the -different acoustic velocities in the two halves of the bar ($v = 1/\sqrt{\rho s^E_{11}}$) on the input side and $1/\sqrt{\rho s^D_{33}}$ on the output side). This can be accounted for in the equivalent circuit, but the complication would make the solution cumbersome without the use of a computer, and this would shed little additional light on the effects of electric load and stray capacitance. Use of the simplified circuit, however, gives slightly inaccurate values for stress. For this reason values of stress are obtained here as previously shown and the equivalent circuit will be used merely to illustrate the effects of electric load and stray capacitance.We shall see later that the difference in velocities leads to other difficulties.

At resonance the Thevenin equivalent values $-2X = X_2$. Since X is a function of frequency and load resistance, and with X a function of frequency, the resonant frequency is a function of load. This is. probably not an important point for contemplated loads.

If we are indeed operating at resonance, the efficiency (neglecting dielectric losses) is given by:

(12) $$c = \frac{R_2}{R_2 + 2R}$$

Going back to our example, let us consider L = 3.0 cm, w = 0.83 cm, t=2 mm so that the volume (2Lwt) is one cubic centimetre. For this case (assuming $Q_M = 500$):

ϕ	=	-6.95×10^{-2} N/V	
coo	=	5.0 pf $(1-k_{33}^2)$=	3.1 pf
R	=	2.9 Nsec/m	
ψ	=	8.65×10^{-3} N/V	

Then consider that the transformer must deliver, through a voltage doubler, 10,000 volts at 100µamperes (one watt) d.c. The actual voltage supplied by the transformer must thus be 5,000 volts peak at 400 µamperes peak; giving $R_L = 1.25 \times 10^7$ ohms. The Thevenin equivalent value for R_2 is thus :

R_2 = 62.5 N sec/m

Unless $R_2 > 2R$, the efficiency is low. The power dissipated in the Thevenin equivalent circuit resistance R_2 is equal to the power delivered to the load.

(13) $$W = 0.5I^2R_2 = 0.5V_L^2 / R_L$$

Here we have also, where I is actually velocity (m/sec)

(14) $$I = V_g (2\phi)/2R + R_2$$

We thus have the input voltage given by:

(15) $$V_g = V_{in} = V_L(2R + R_2) / \sqrt{R_2 RL}\, 2\phi$$

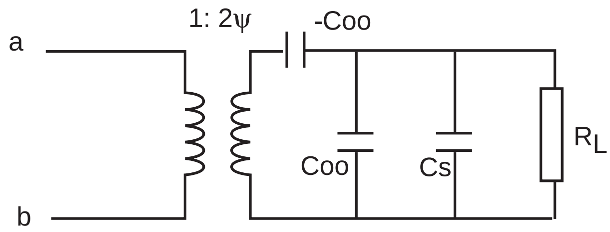
For the case at hand V_{in} = 87. 5 volts. This is about a factor of three greater than the input voltage required to obtain the same output voltage into an open circuit. Thus about three times the input voltage is required to obtain the same stress. This is perhaps best interpreted as a reduction of effective mechanical Q. Unless it is considerably reduced, of course, no power can be delivered to a load. Essentially the effective Q is reduced by the ratio $R_2 / 2R \approx 10\ 8$. Internally dissipated power values listed in Section 2 apply here also, since $R_L^2 >> X_o^2$. Under these idealized conditions, one should expect this transformer, of one cm³ volume, to have the following performance.

1. Output power one watt at 5,000 volts.
2. Operating frequency 31.5 kHz
3. Dissipated power $(P_{DM} + P_{DE}/2)$ about 0.24W
4. Efficiency ≈ 82%
5. Input voltage about 90 volts
6. Temperature rise with no thermal transfer or radiation is about 0. 06°C/sec = 3.6°C/minute.
7. Dynamic stress about 1000 psi (space average).

Note that the simple efficiency equation one can readily obtain from the Thevenin equivalent circuit, namely $e = R_2/(2R + R_2)$ is not correct, since it fails to account for dielectric losses. These are accounted for in the list above.

4. ANALYSIS WITH ELECTRIC WAD AND STRAY OUTPUT CAPACI-TANCE; ZERO IMPEDANCE GENERATOR DRIVE

The picture presented in Section 3 is, however, beclouded by the effects of stray capacitance across the output, since the output capacitance C_{oo} in the equivalent circuit is only 3.1 pf for the example at hand. Again lumping dielectric losses in R_L, but accounting for stray capacitance C_s, the output section of the equivalent circuit is as shown below:



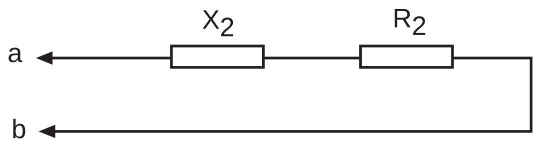
Setting

(16) $$-1/(j\omega C_{oo}) = jX_a$$ and

(17) $$1/(j\omega C_{oo} + C_s) = -jX_b$$

j. w c 00 = j X a and
jw (C +C) = j X

it is readily shown that the following equivalent circuit holds.



Here:

(18) $$R_2 = \frac{X_b^2 R_L 4\psi^2}{R_L^2 + X_b^2}$$

(19) $$X_2 = \frac{X_a R_L^2 + X_a X_b^2 - X_b R_L^2}{R_L^2 + X_b^2} 4\psi^2$$

These may be inserted into the Thevenin equivalent as shown previously, now taking account of stray capacitance effects. We can ignore the change in X_2 due to stray capacitance for our purposes. but it should be noted that the change in resonant frequency (for $-2X = X_2$) due to stray capacitance as well as resitive load can (and must) be accounted for.

If we now go back to our previous example and let $C_s = C_{oo} = 3.1pF$, we can readily obtain insight into the effects of stray capacitance on operating characteristics.

For the example at hand $R_L >> X_b$, Eq. (18) simplifies to

$$(20) \quad R_2 = \frac{X_b^2 4\psi^2}{R_L} = 12.5 \text{ N sec/m}$$

Here X_b is just one-half its previous value (Section 3, where loading effects of R_L were considered, but not stray capacitance), so R_2 is less than onefourth its previous value. Application of Eq. (15) leads here to a new value for $V_g = V_{in}$. It should be noted that R in Eq. (15) is a function of Q_M, which is in turn a function of stress. It turns out that our previous value for R is fairly close and may be used again, since R_2 is still several times larger than R we obtain

$V_g = V_{in} = 53$ volts

The velocity given by Eq. (14) is thus 0.40 m/sec compared to 0.20 m/sec for the same load and zero stray capacitance. Since stress is proportional to velocity (frequency constant), the stray capacitance has in this case doubled the stress required to obtain the necessary output voltage, even though it has decreased the required input voltage. For loads of the type discussed here ($R_L^2 >> X_{oo}^2$), stray capacity affects stress level in the
L 00
following manner.

00 S
(21) Tstray cap. = Tno stray cap. coo
The required input voltage is decreased because the effective Q is increased. We shall see that this results in lowered efficiency (already obvious from Thevenin equivalent values).

Carrying through on our specific example, we see that the average dynamic stress is now 2, 000 psi. We see that Q M is still about 500 at this stress level, so the mechanically-,, dissipated power PDM = 0. 37 watts/ cm 3 We now find the characteristics of the transformer as follows (dielectric losses unchanged).

1. Output power one watt at 5000 volts

2. Operating frequency 31. 5 kHz

3. Dissipated power (PDM + PDE) about 0.50 watts

4. Efficiency ≈ 6 7%

5. Input voltage 53 volts

6. Temperature rise with no heat transfer or radiation about 0. 15°C/sec = 9°C/minute

7. Average dynamic stress about 2000 psi

8. Stray capacitance = 3.1

5. DISCUSSION OF RESULTS

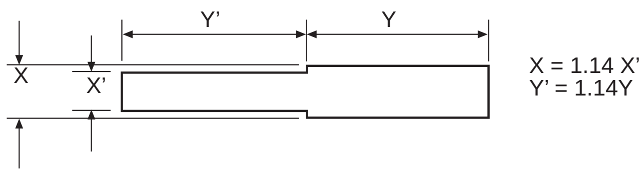
Results on experimental transformers have in general been disappointing. It has been found that heating effects are much more severe than calculated, and nonlinearity of the V_{in} vs. V_{out} characteristic is much worse than calculated on the basis of the Q_m vs. stress curve of Fig. 1. In an effort to shed some light on these results, production PZT-8 rings were run for one hour continuously at two different stress levels and at two different temperatures, as follows:

Temp, °C	Peak Dynamic Stress, psi	
25	1500	
70	1500	All at 31. 7 kHz
25	3000	
70	3000	

In no case was heating severe, and there was no degradation in Q_M values from those shown in Fig. 1.

It is thus necessary to ask in what manner, other than the sinusoidal stress distribution, the operating conditions of the transformer differ from those of the rings. We first note that there is significant dielectric loss only in the transformer (output section). Yet in operation the input section of the transformer gets much hotter than the output section.

We are forced to conclude that in all probability operation of the transformer is severely degraded by acoustic mismatch between the output and input sections of the transformer. This mismatch apparently results in a much higher average stress in the input section than in the output section. A thorough analysis well beyond the scope of this report would be required in order to provide an analytical basis for this conclusion. The mismatch in acoustic impedances is about 14% for PZT-8. This can be compensated to a certain extent by adjustment of cross-sectional areas and lengths of the two sections as illustrated below.



We are therefore preparing some modified transformers in the hope that this will eliminate the tendency for stress concentration at the input end.

All in all, the preceding analysis must be considered evidence that the transformer can be made to work if stress concentration can be eliminated. It must be emphasised, though, that we have not obtained satisfactory performance in actual transformers. Heating has been so severe that soldered connections have often melted. Since care has. been taken to provide supports free from mechanical constraints, it is not felt that external damping is a factor. Mechanically dissipated heat depends upon the actual value of Q_M, including external as well as internal damping, but only the latter contributes directly to temperature rise in the active ceramic.

Failure to obtain predicted characteristics with ceramic transformers is thought to be due to one or a combination of the following four factors.

1. Stress concentration due to poor acoustic match
2. Inferior PZT-8 material.
3. Introduction of external damping at, high stress levels
4. Stray capacitance.

The first factor may well be the most important, but we have no definite results. Some poor performance data can be traced to inferior PZT-8 material. We have so far had only one satisfactory batch of PZT-8 elements which could be rebuit as transformers. Since these were rings it was necessary to use them as ring transformers, and mechanical failure (radial cracking) occurred at about 5,000 volts on a 15 kHz ring. In general we have found cracking a much more severe problem with rings than with bars. The additional difficulty of frequency adjustment also favors the bar approach.

In the experimental set-up used in this work external damping and stray capacitance are not thought to be factors. The effects of stray capacitance nevertheless constitute a severe limitation on utility of the device.

A frequency intermediate between the fundamental and first overtone of the horizontal scan frequency would be preferred from the standpoints of acoustic noise, required stress level, and power dissipation, but the economic view probably requires first overtone operation. Only if heat rise can be held low can the horizontal scan frequency be used directly.

A real advantage of a device of this kind would be involved in an application where its unique frequency selective characteristics would be required. Even at best, as in the idealized case analyzed here, efficiencies are much lower than those of coil type transformers in this frequency range and mechanical stresses are orders of magnitude higher.

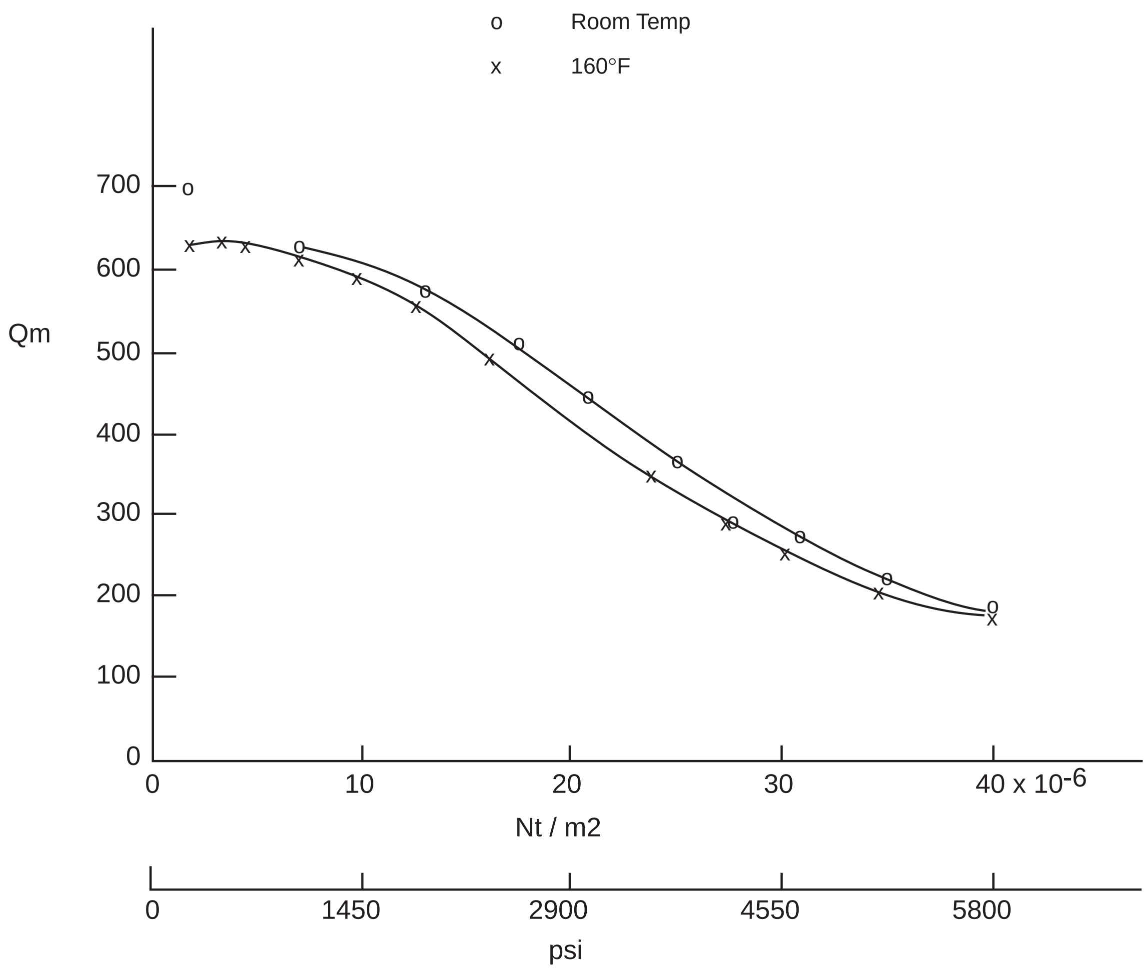


Fig. 1 PZT-8: Qm vs Stress at Two Temperatures