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USEFUL RELATIONSHIPS FOR CIRCULAR CERAMIC BENDER BIMORPHS

INTRODUCTION

A collection of useful relationships applying to circular ceramic bender Bimorphs(g) is presented. These are shown for two common mountings and three typical loading conditions. As a simply supported (circumferentially) plate one may drive the element with either a concentrated load at the center or with a uniformly distributed load over the entire surface. It is also of interest to consider the latter load conditions for a centrally supported plate. These three cases are included in the following work.

1. DEFINITION OF TERMS

Many terms will be encountered in this report. These are defined below.

A. Equivalent Circuit Constants

c_e	Free electrical capacitance
c_e'	Blocked electrical capacitance
N	Transducer ratio V_{oc}/F
N_a	Transducer ratio V_{oc}/P
N′	Transducer ratio F_b/V
N_a'	Transducer ratio P_b/V
C_m	Compliance (open circuit conditions)
C_v	Volume,compliance (open circuit conditions)
C_m'	Compliance (short circuit conditions)
C_v'	Volume compliance (short circuit conditions)
M, M′	Effective Mass
I_a, I_a'	Equivalent inertance

B. Piezoelectric, Dielectric, and Elastic Constants

k_{31}	Transverse coupling factor
k_b	Bending coupling factor
g_{31}	Piezoelectric constant relating field developed to applied stress
d_{31}	Piezoelectric constant relating strain developed to applied field
$\varepsilon^T_{33}/\varepsilon_o$	Dielectric permittivity (free)
$\varepsilon^S_{33}/\varepsilon_o$	Dielectric permittivity (blocked)
Y^E_{11}	Young's Modulus short circuit conditions
Y^D_{11}	Young's Modulus open-circuit conditions
ρ	Density

C. Dimensions

a	Radius of Bimorph
t	Overall thickness of Bimorph (Assumed to be twice the thickness of one plate of Bimorph)
r_o	Radius of support of centrally supported condition

D. Applied and Developed Parameters

V	Applied voltage
V_{oc}	open circuit voltage
I	Applied current
I_{sc}	Short circuit current
Q	Applied charge
Q_{sc}	Short circuit charge
F	Applied force
P	Applied pressure
F_b	Blocked force
P_b	Blocked pressure
U	Applied velocity
U_v	Applied volume velocity
U_f	Free velocity
U_{vf}	Free volume velocity
D	Applied displacement
D_v	Applied volume displacement
D_f	Free displacement
D_{vf}	Free volume displacement

II. EQUIVALENT CIRCUITS AND CIRCUIT CONSTANTS

The parameters shown in Table I are the circuit constants applying to the electromechanical equivalent circuits shown in (a) of Figure 1. The alternative representations shown in (b) of this figure are also of interest. The conversion of the constants of Table I is facilitated by the use of the interrelationships shown in Figure 1.

Elements of electrical and mechanical dissipation are neglected for simplicity. However, their presence have a profound affect on practical transducer design. This is especially true for motor applications under high electrical drive. This simplification does riot, however, affect the circuit constants shown in Table I. The input-output relationships to be shown later are, of course, affected and proper allowance should be made for this.

The derivations (which are definitely non rigorous) leading to the parameters (which therefore are only approximate) of Table I are not included here, but are based on ideally mounted elements. Liberal use is made of references1,2 for appropriate stress functions and other mechanical properties. Second order correction factors such as the effect of brass vanes (usually found in typical rectangular benders) and unstressed portions of the mounted element are neglected. A previous Engineering Memorandum3 includes these for rectangular benders and reference may be made to this report for their form.

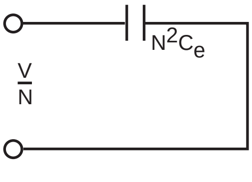
III. **ELECTRICAL AND-MECHANICAL OR ACOUSTICAL OUTPUTS**

The appropriate use of the circuits shown in Figure 1 leads to the input-output relationships shown in Tables II and III.

Operation below resonance is assumed and the ideally mounted elements are unloaded. Thus, to obtain free displacements and velocities, the mechanical terminals are shorted, while for the determination of blocked force and pressure, they are open circuited. Similarly, short circuit currents and charges are obtained by shorting the electrical terminals and obviously, open circuit conditions apply to obtain-open circuit voltages.

At resonance the free displacements and velocities are increased approximately by Qm the mechanical quality factor. This is also true of the open circuit voltage and short circuit current and charge when driven by a force at resonance. Thus a knowledge of Om is required and this, in effect, defines the mechanical loss factor.

As an example of the procedure used to determine the relationships of Tables II and III, consider circuit (a) of Figure 1 with an applied voltage V. At low frequencies and for free vibration, the circuit reduces very simply to

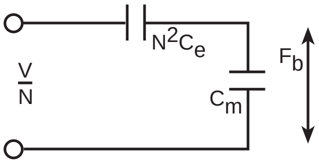


Transposing electrical to mechanical side and shorting mechanical terminals.

In analogous fashion to electrical charge on a capacitor where

Q = CV, we have $D_f = \frac{V}{N} N^2C_e = NC_eV$

Again at low frequencies and for a blocked force, the mechanical terminals are left open and the circuit becomes



$$\text{Thus } F_b = \frac{N^2c_e}{N^2c_e + c^m} \cdot \frac{V}{N}$$

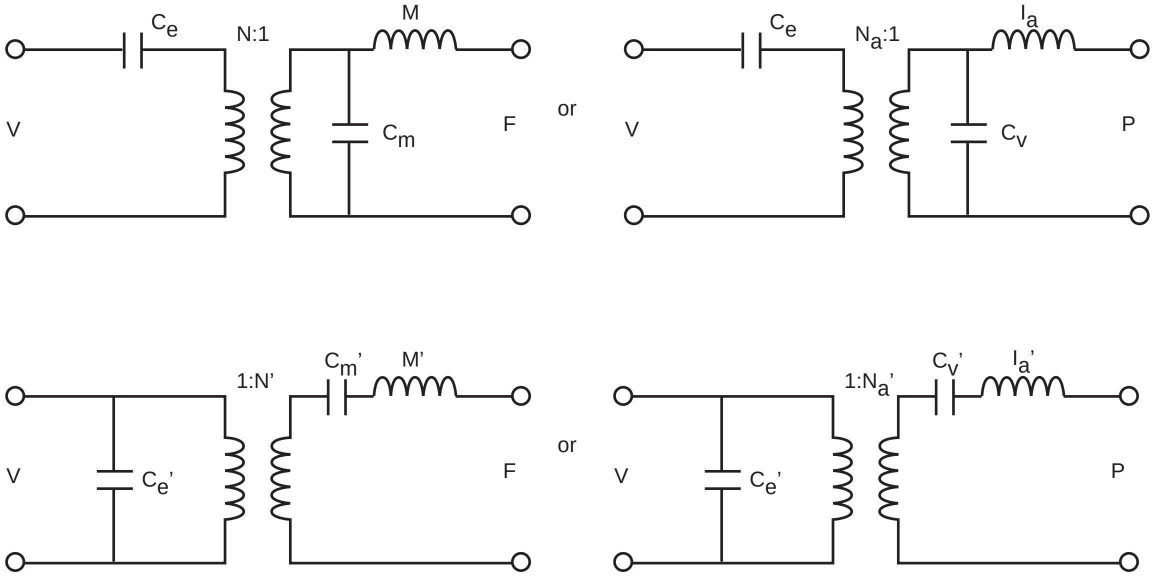
$$= \frac{Nc_eV}{N^2c_e + c^m}$$

Proceeding on a similar basis, we can set up Tables II and Ill summarizing the relationships.

IV. TABLE OF PROPERTIES OF PZT®

The relationships shown in the aforementioned tables are functions of piezoelectric, dielectric, and elastic constants. Several compositions may be used in the fabrication of circular Bimorphs although only PZT-5B and PZT-5H are considered as standard. The important properties are listed in Table IV for these compositions. It should be noted, however, that those shown for the latter two materials apply for so-called single plate elements. In typical Bimorphs, very thin sheet material is used which along with “Bimorphing” losses lead to effective constants substantially different than shown here. For example, as much as 20-25% reduction in effective dielectric constant is experienced with thin (.021" and .024") PZT-5B Bimorphs. With PZT-5H, this loss is even greater. Losses in g-constants are less significant but the effective d-constants display similar reductions. The effective values of elastic constants are marginally lower than those shown here.

Fig. 1 Electromechanical Equivalent Circuits



$$C_e' = \frac{C_e C_m}{N^2 C_e + C_m} = \frac{C_e C_v}{Na^2 C_e + C_v}$$

$$C_m' = N^2 C_e + C_m$$

$$N' = \frac{N C_e}{N^2 C_e + C_m}$$

$$C_v' = N_a{}^2 C_e + C_v$$

$$N_a' = \frac{N_a C_e}{N_a{}^2 C_e + C_v}$$

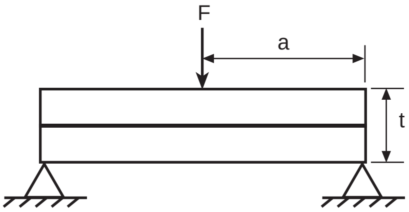
$$M = M'$$

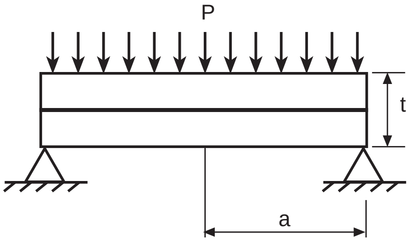
$$K^2_b = \frac{N^2 C_e}{N^2 C_e + C_m} = \frac{N_a{}^2 C_e}{Na^2 C_e + C_v}$$

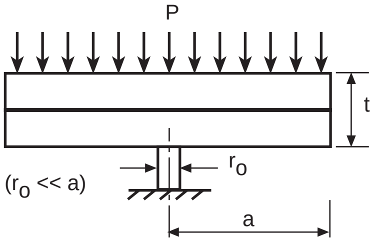
$$I_a = I_a'$$

Table I. Approximate Relationships for Equivalent Circuit Constants

Case	Transducer Ratio N v/n	Na v n/m2	Capacitance Ce farads	Compliance Cm m/n	Acoustic Cv m3/ n/m2	Eff Mass M kg	Equiv Inert Ia kg/m4	Res Freq fr Hz	Bend Coupling kb
A	$\frac{3}{2\pi} g_{31} \frac{1}{t}$		$K^T_3 \varepsilon_o \pi \frac{a^2}{t}$	$\frac{1}{2Y^D_{11}} \frac{a^2}{t^3}$		$\frac{1}{4} \rho \pi a^2 t$		$0.25 \frac{t}{a^2} \sqrt{\frac{Y^E_{11}}{\rho}}$	$1.1 k_{31}$
B		$\frac{3}{4} g_{31} \frac{a^2}{t}$	$K^T_3 \varepsilon_o \pi \frac{a^2}{t}$		$\frac{1}{Y^D_{11}} \frac{a_6}{t^3}$		$0.4\rho \frac{t}{a_2}$	$0.25 \frac{t}{a^2} \sqrt{\frac{Y^E_{11}}{\rho}}$	$1.2 k_{31}$
C		$\frac{3}{4} g_{31} \frac{a^2}{t}$	$K^T_3 \varepsilon_o \pi \frac{a^2}{t}$		$\frac{1.6}{Y^D_{11}} \frac{a_6}{t^3}$		$0.5\rho \frac{t}{a_2}$	$0.18 \frac{t}{a^2} \sqrt{\frac{Y^E_{11}}{\rho}}$	k_{31}







Transducer Ratio applies for series connected Bimorph. For paraller Bimorphs multiply by 0.5
Capacitance applies for series connected Bimorph. For paraller Bimorphs multiply by 4

Table II

Electrical Input	Mechanical or Acoustical Outputs		
V volts	$F_b = \frac{NC_eV}{N^2C_e + C_m}$	$U_f = j\omega NC_eV$	$D_f = NC_eV$
	$P_b = \frac{N_aC_eV}{N^2C_e + C_m}$	$U_{vf} = j\omega N_aC_eV$	$D_{vf} = N_aC_eV$
I Amperes	$F_b = \frac{NI}{j\omega C_m}$	$U_f = NI$	$D_f = \frac{NI}{j\omega}$
	$P_b = \frac{N_aI}{j\omega C_v}$	$U_{vf} = N_aI$	$D_{vf} = \frac{N_aI}{j\omega}$
Q Coulombs	$F_b = \frac{N_aQ}{C_m}$	$U_f = j\omega NQ$	$D_f = NQ$
	$P_b = \frac{N_aQ}{C_v}$	$U_{vf} = j\omega N_aQ$	$D_{vf} = N_aQ$

TABLE -IV

Property	**PZT-4***	**PZT-5A**	**PZT-5B**	**PZT-5H**
k_{31}	-33	.34	.34	.39
g_{31} (10^{-2} volt meters/newton)	-11.1	-11.4	-10.5	- 9.0
d_{31} (10^{-12} meters/volt)	-123	-171	-185	-275
$\varepsilon^T_{33} / \varepsilon_o = K^T_3$	1300	1700	2000	3400
Y^D_{11} (10^{10} newtons/meter^2)	9.2	6.9	6.9	7.2
Y^E_{11} (10^{10}newtons/metre^2)	8.2	6.1	6.1	6.1
ρ (10^3 kg/m^3)	7.6	7.7	7.5	7.5

*Because aging is somewhat more severe for PZT-4 than with the other materials listed, these data apply for material aged approximately 10 days. Beyond this point, additional aging becomes less significant.