

Visit the  
Morgan Electro Ceramics Web Site

[www.morgan-electroceramics.com](http://www.morgan-electroceramics.com)

## GENERAL DESCRIPTION OF PIEZOELECTRIC TRANSFORMERS

by D. A. Berlincourt

### 1. INTRODUCTION

A piezoelectric transformer as described here is a three or four terminal device with one terminal pair connected to the input, the other to the output. Various configurations can be used and several are described briefly in Section 1.1. Only at resonance is high efficiency obtained, so the piezoelectric transformer could also be considered a filter. Indeed it is; a square wave drive with the repetition rate equal to the resonant frequency will produce a secondary voltage and current that is sinusoidal at the repetition rate frequency.

A piezoelectric transformer has characteristics much different from those of usual coil transformers. For instance, both a short-circuited and an open-circuited secondary look like a short circuit to the generator. Maximum efficiency occurs with the secondary load which makes the input impedance at the primary a maximum. This is when  $R_L$ , the load impedance, is equal to the impedance of the clamped capacitance of the output section. The operating characteristics are described in Section IV, based on the best piezoelectric material for this application (PZT-8) described in Section III.

Piezoelectric transformers offer certain advantages over conventional coil transformers. For high voltage low current operation, they are generally lighter and there is no fire hazard. In some cases, the peculiar regulation and narrow band (filter) nature of the device can be useful. It will always be necessary, however, to evaluate the design of a piezoelectric transformer very carefully for any given application. The design process is relatively complex; some aspects are described briefly in Section IV. Section II describes some of the many possible configurations and Section III shows characteristics of the composition (PZT-8) best suited for high amplitude piezoelectric transformers.

### II. CONFIGURATIONS FOR PIEZOELECTRIC TRANSFORMERS

Figure 1 illustrates some of the many configurations which may be utilized. Types 1, III, V and VI give relatively high step-up; Types IV and VII moderate

step-up; and Type II no step-up (unless it is Impedance mismatched) Choice of various transformer configurations, specific dimensions and dimensional ratios affects input and output impedances and frequency as well as voltage step-up. All configurations shown other than Type VII operate about equally well at the first overtone as at the fundamental. Mounting at nodal points is different depending upon overtone or fundamental operation. Multiple secondaries may also be provided with configurations similar to those illustrated in Figure 1.

It is, however, not clear that multiple secondary configurations are practical, since there is interaction between loads. Analysis may be carried out using minor extension of methods presented in this report.

Some of the design formulas for transformers of Types I, III, III and V are listed in Section IV.

### III. CHARACTERISTICS OF PZT-8 OF INTEREST FOR APPLICATION IN PIEZOELECTRIC TRANSFORMERS

Both dielectric and mechanical-losses affect performance of piezoelectric transformers. Both affect efficiency and power dissipation, but mechanical losses are by far the most important. Mechanical losses also affect linearity and voltage step-up at high amplitude, especially with the type of impedance mismatch required for high voltage step-up (see Section. IV). PZT-8 has remarkably low losses as shown in Figure 2; mechanical losses do not increase rapidly with mechanical stress and dielectric losses remain low at high electric drive. The well-known mechanical quality factor  $Q_M$  is equal to  $1/\text{Tan}\delta_M$  in Figure 2.

Table I lists parameters of PZT-8 required for analysis of performance of piezoelectric transformers shown in Figure 1. These values are utilized in Section IV. which summarizes analytical procedures.

TABLE 1  
PARAMETERS OF PZT-8

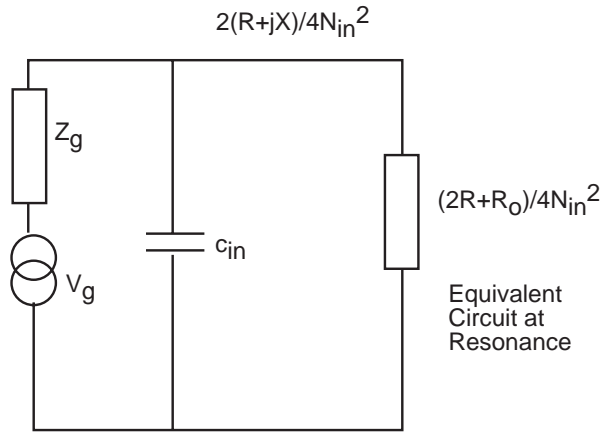
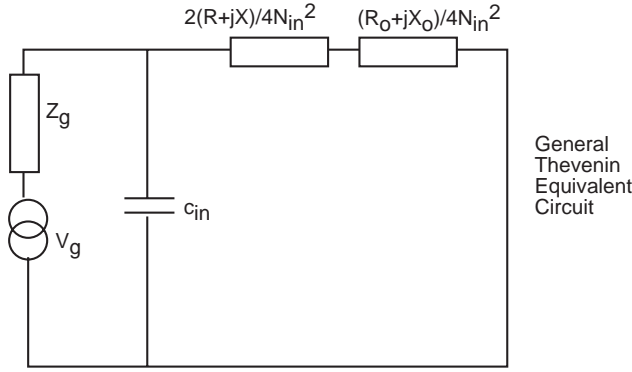
$$\begin{aligned} k_{31} &= k_{31}^1 = -0.30 \\ k_{33} &= k_{33}^1 = 0.64 \\ k_p &= -0.51 \\ k_{33}^W &= 0.60 \\ S_{11}^E &= 11.5 \times 10^{-12} \text{m}^2/\text{N} \\ \sigma_P^E &= 0.295 \\ S_{33}^D &= 8.0 \times 10^{-12} \text{m}^2/\text{N} \end{aligned}$$

$$\begin{aligned}
S_{33}^E &= 13.5 \times 10^{-12} \text{ m}^2/\text{N} \\
S_{13}^E &= 4.7 \times 10^{-12} \text{ m}^2/\text{N} \\
\sigma_m^D &= 0.253 \\
\varepsilon_{33}^T/\varepsilon_o &= 1000 \\
d_{31} &= -97 \times 10^{-12} \text{ m/V} \\
d_{33} &= 225 \times 10^{-12} \text{ m/V} \\
g_{31} &= -11.0 \times 10^{-3} \text{ Vm/N} \\
g_{33} &= 25.5 \times 10^{-3} \text{ Vm/N} \\
\rho &= 7.5 \times 10^3 \text{ kg/m}^3 \\
Q_M &= 1/\tan \delta_M \text{ (see Fig. 2)} \\
O_E &= 1/\tan \delta_E \text{ (see Fig. 2)}
\end{aligned}$$

Figures 3, 4, and 5 summarize the temperature behaviour of some of the important parameters in Table I.

#### IV. PERFORMANCE OF PIEZOELECTRIC TRANSFORMERS - EQUIVALENT CIRCUIT ANALYSIS

The general equivalent circuit for a piezoelectric transformer is shown below. The simplification for operation at resonance is shown also. Operation at or very close to resonance is a virtual necessity.



Parameters for transformers of Types 1, 11, III, and V are listed in Table 11.

TABLE II

|      | Type I  | Type II   |
|------|---|---|
| Cin  | $\varepsilon_{33}^T(1-k_{31}^2)lw/2t$                     | $\varepsilon_{33}^T(1-k_{31}^2)lw/2t$               |
| Cout | $\varepsilon_{33}^T(1-k_{31}^2)2wt/l$                     | $\varepsilon_{33}^T(1-k_{31}^2)lw/2t$               |
| Nin  | $wd_{31}/s_{11}^E$  | $wd_{31}/s_{11}^E$                                  |
| Nout | $2wtd_{33}/ls_{33}^E$                                     | $wd_{31}/s_{11}^E$                                  |
| v    | $1/(2\sqrt{\rho s_{11}^E})+1/(2\sqrt{\rho s_{11}^E})$     | $1/(2\sqrt{\rho s_{11}^E})$                         |
| fr   | $v/2l^{**}$   | $v/2l^{**}$   |
|      | Type III  | Type V  |
| Cin  | $\varepsilon_{33}^T(1-k_p^2)lw/2t$                        | $\varepsilon_{33}^T(1-k_{31}^2)l_a w/t$             |
| Cout | $\varepsilon_{33}^T(1-k_{33}^w)(1-k_{31}^2)2lt/w$         | $4\varepsilon_{33}^T(1-k_{31}^2)wt/l_b$             |
| Nin  | $[ld_{31}(1+\sigma_p^E)]/s_{11}^E[1-\sigma_p^E]$          | $wd_{31}/s_{11}^E$                                  |
| Nout | $2k_{33}^w lt/w[g_{33}g_{31}(s_{13}^E/s_{11}^E)]$         | $2(wt/l_b)(d_{33}/s_{33}^E)$                        |
| v    | $1/[2s_{11}^E(1-\sigma_p^E)]+1/[2s_{33}^D(1-\sigma_m^D)]$ | $1/(2\sqrt{\rho s_{11}^E})+1/2\sqrt{\rho s_{33}^D}$ |
| fr   | $v/2w^{**}$   | $v/l^{**}$  |

$$\begin{aligned}
R &= (\pi/QM)rvwt^* \\
R_o &= 4N_{out}2R_L/[1+(\omega RR_L(C_{out}+C_s))^2] \\
V_L/V_{in} &= 2N_{in}\sqrt{R_o R_L}/(2R+R_o) \\
R_{in} &= (2R+R_o)/4N_{in}^2 \\
\sigma_p^E &= -s_{12}^E/s_{11}^E \quad ; \quad \sigma_m^D = -s_{13}^D/\sqrt{s_{33}^D s_{11}^D}
\end{aligned}$$

\*\*Multiply by 2 for operation at first overtone.  
\*Multiply by 1/2 for operation at first overtone.

$R_L$  = load resistance.  $C_s$  = capacitance of load.

Bars;  $l, w, t$  are length, width and thickness.

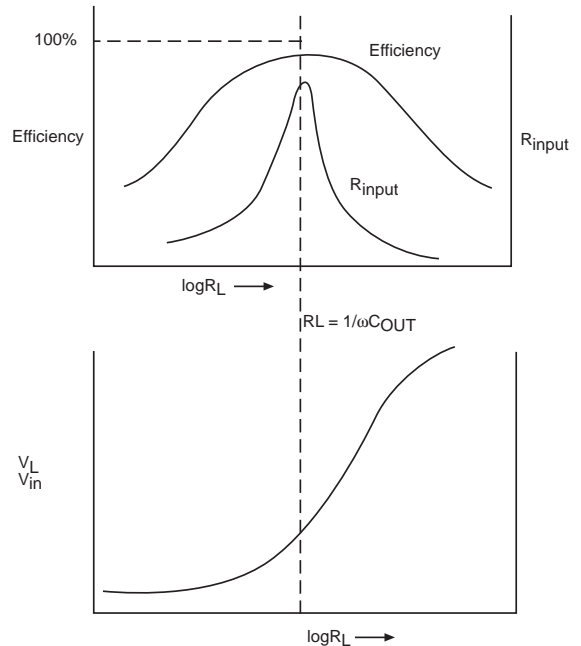
Ring;  $l$  = average circumference,  $l_a$  and  $l_b$  lengths of input and output respectively;

$w$  = height

$t$  = wall thickness.

mks units throughout.

The typical operating characteristics of a piezoelectric transformer at resonance are summarized in the sketches below.



Exact performance depends upon dimensional factors as well as material parameters, but behaviour always follows this pattern. Resonant frequency depends upon  $R_L$ , but this is not a factor unless  $R_L$  varies drastically. Efficiency and input resistance peak, the former broadly and the latter sharply when  $R_L = 1/\omega C_{out}$ . Efficiency at this point is generally over 90%. Typically an impedance mismatch of 100 to 1 leaves efficiency above 40% and an impedance mismatch of 15 to 1 leaves efficiency above 80%, but this is dependent upon piezoelectric coupling and thus the specific configuration. The sacrifice is thus somewhat more for the lower coupling configurations (Types 11 and VII) than for the others.

Impedance mismatches of 100 to 1 or 15 to 1 on the high side with a typical transformer of Type I raise voltage transformation from 10 to Over 200 or about 100 respectively.

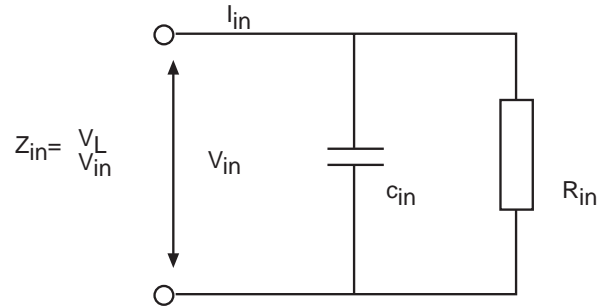
Calculated performance characteristics for transformers of Types I and V are illustrated in Figures 6 through 8. More nearly exact calculation would require consideration of the variation of mechanical and dielectric loss factors with amplitude (see appendix). This can be done considering the primary voltage held constant at a given value, in which case electric field in the output section and mechanical stress vary with the load  $R_L$ . It can also be done considering the secondary voltage held constant with varying load  $R_L$ , which requires changes in primary voltage; account would then be taken of changes in stress with varying load. Curves in Figures 6 through 8 were drawn neglecting dielectric losses and considering the mechanical Q constant at 500. The dashed curve for efficiency in Figure 6 allows for dielectric loss, significant only in the output section, considering the output voltage held constant at 5 kV.

Figure 7 shows improvement in regulation and decrease in overall circuit efficiency produced for the transformer of Figure 6 by varying the generator impedance. For high step-up, regulation is always poor. As noted previously, however, only a relatively small sacrifice in efficiency is required to achieve high step-up by impedance mismatch on the high side.

The effects of stray or load capacitance  $C_s$  are accounted for using the relations in Table II. Effects are generally relatively small for all configurations shown in Figure 1 except Type I and Type V unless the load contains significant capacitance. Stray capacitance, if significant, limits output power because it increases the required dynamic stress level for a given output electric field. These deleterious effects are roughly proportional to the ratio  $C_s/C_{out}$

Figure 9 shows calculated load characteristics for a particular PZT-8 Type II transformer. Here dielectric and mechanical losses are included. Figure 10 shows measured load characteristics for a Type IV transformer having dimensions noted. The upper curve shows the efficiency.  $|Z_{in}|$  is the absolute value of the

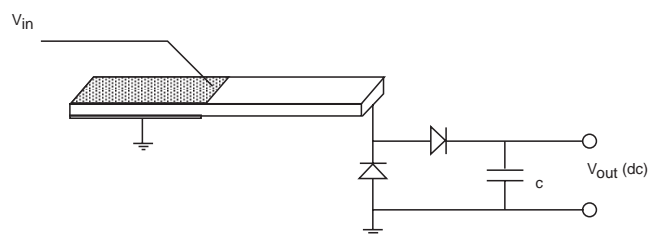
input impedance.



$$|Z_{in}| = \frac{R_{in}}{\sqrt{(1 + \omega^2 C_{in}^2 R_{in}^2)}}$$

The phase angle  $\theta$  is the angle by which  $I_{in}$  leads  $V_{in}$ ;  $\theta = \arctan R_{in} \omega C_{in}$ . The resonant frequency changes significantly only in the region near  $R_L = 1/\omega C_{out}$ . Figure 11 shows  $I_L$  Vs  $V_L$  for constant  $V_{in}$ . The two ranges of interest for operation of gaseous discharge lamps are roughly divided by the dashed line with slope equal to  $1/\omega C_{out}$ . In the low current range, regulation is extremely poor. The high starting voltage is provided in this range. Once the transformer is excited, the period of roughly Q cycles is required for build-up of secondary voltage; initiation of the arc with a fluorescent lamp would limit build-up to the 300-800 volt range so that mechanical stress remains within safe limits. To eliminate completely the possibility of mechanical damage, a 10,000 ohm resistance could be placed across the secondary without significantly affecting efficiency in the optimum load range. In the high current range, regulation is quite good, but an increase in 'L causes a decrease in  $V_L$  ( $V_{in}$  constant). This assures proper regulation for the negative resistance range of gaseous discharge lamps. Dimming can simply be accomplished by changing  $V_{in}$  a decrease in  $V_{in}$  causes a decrease in  $I_L$  rather than a decrease in  $V_L$ .

When dc power is required from the secondary of a piezoelectric transformer, the rectifier circuit shown below is used.  $C_{out}$  of the transformer is nearly always sufficient to provide one of the capacitors for the doubler, and in many cases the second capacitance C is provided by capacitance of the load or the high voltage lead. The high voltage diodes should have relatively low capacitance; selenium stacks work well.



## V. APPLICATIONS

It is not possible to define a group of standard piezoelectric transformers. They lack the universality of coil transformers. Variations in dimensions and electrode configuration, however, permit a wide range of operating characteristics and allow adjustment for specific applications. In general, however, very high step-up ratios are obtained only with transformers matching high output impedances (high efficiency operation and only fair regulation) or with significant mismatch on the high side (relatively low efficiency and poor regulation). Good regulation is obtained only, with impedance mismatch on the low side and in such case relatively little step-up is obtained.

Two major application areas are as follows :

(1) Starting and ballast operation of gaseous discharge lamps. Advantages are small size, light weight, high efficiency, no fire hazard, very high starting voltage, dimming capability, silent operation. While cost for an oscillator circuit or other high fixed frequency drive may be a deterrent, this should be weighed against system improvement through weight reduction, operational flexibility and greater reliability.

(2) High voltage applications such as TV or other CRT systems. Advantages generally correspond with those listed above. Regulation is poor, but can be achieved at moderate additional cost by adjustment of drive level through a feedback arrangement or by small frequency adjustments. One of the most important advantages is absolute elimination of fire hazard.

### V1. APPENDIX; CALCULATION OF DIELECTRIC AND MECHANICAL LOSS POWER

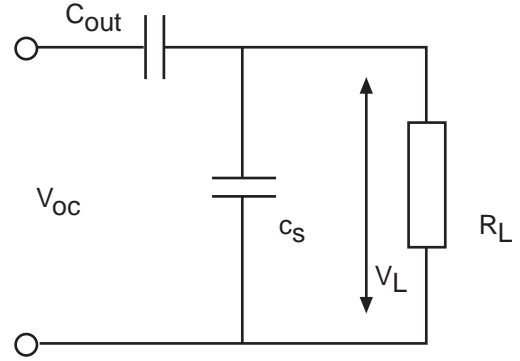
Dielectric loss power is generally significant only in the output section for usual load conditions. Since dielectric loss does not significantly affect voltage step-up, it is necessary merely to use the calculated output voltage  $V_L$ . This is changed to units of electric field and the loss factor  $\tan \delta_E$  is found in Figure 2b, for PZT-8. The dielectric loss power  $P_{DE}$  (watts) in the output section is given by:

$$P_{DE} = \frac{\omega V_L^2 C_{out} \tan \delta_E}{1 - k^2}$$

The coupling factor of the output section is required because  $\tan \delta_E$  refers to the low frequency or free dielectric constant, while  $C_{out}$  is a partially clamped dielectric constant. The coupling factor for each of the configurations of Table II is that shown in the expression for  $C_{out}$ . Note that in this expression  $\tan \delta_E$  is a function of  $V_L$ .

The mechanical loss power  $P_{DM}$  is accounted for in the equivalent circuits of Section IV. To account for changes of the mechanical  $Q$  (equal  $1/\tan \delta_M$ ) with stress amplitude, however, the value for  $R$  in Table II must be adjusted for stress. This is not simple, because the value of  $Q_M$  affects voltage step-up and

thus mechanical stress, particularly with  $R_L > 1/\omega C_{out}$ . It can, however, be accomplished starting with an estimated value of  $Q$ . Once the output voltage has been calculated for a specific load condition, the stress can be calculated approximately using the equivalent circuit below.



The voltage  $V_{OC}$  is given approximately by:

$$V_{OC} = g T_{ave} l_o$$

where  $g$  is the piezoelectric constant of the output section ( $g_{33}$  for Types I and V,  $g_{31}$  for Types II and VII,  $g_{33} - g_{31} (S_{13}^E / S_{11}^E)$  for Types III and VI, and  $2g_{31}$  for Type IV),  $T_{ave}$  is the average stress ( $N/m^2$ ), and  $l$  is the length between output terminals. From the circuit above, determine  $a$ , where

$$|V_L| = \alpha |V_{oc}|,$$

and thus

$$T_{ave} = V_L / \alpha g l_o$$

Using this value of stress  $Q = 1/\tan \delta_M$  is determined from Fig. 2a. If necessary, a new voltage step-up may then be calculated after derivation of a new value for  $R$  (Table II). Then the expressions above may be used to determine a new or corrected value of  $T_{ave}$ . Finally the value of  $P_{DM}$  may be determined using the equivalent circuit of section IV.

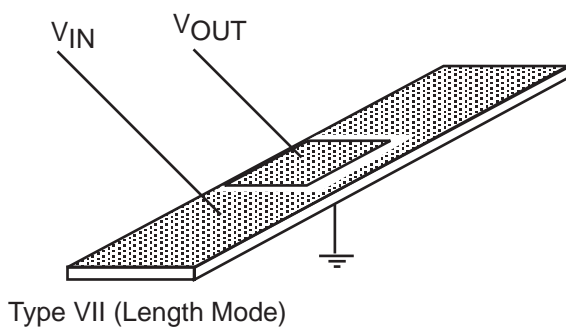
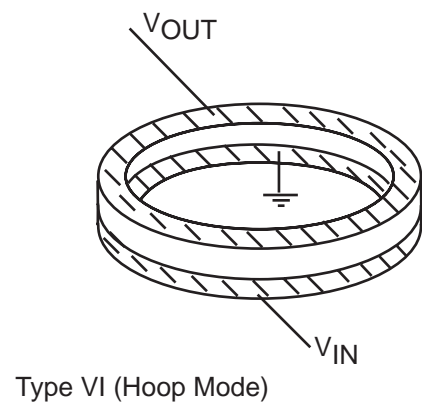
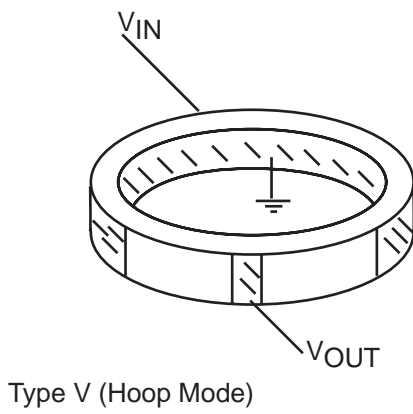
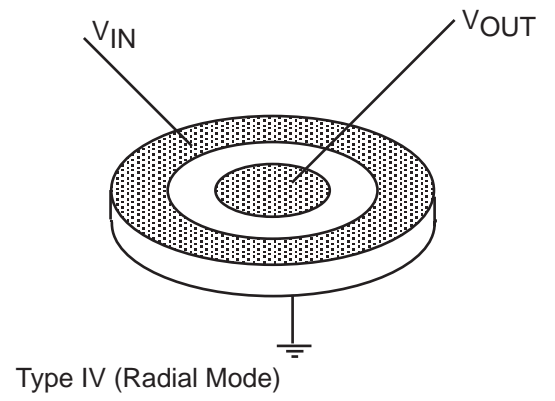
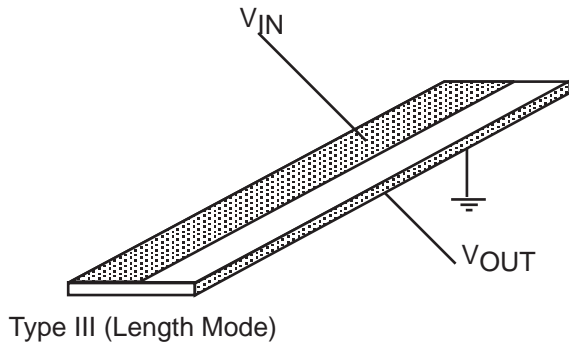
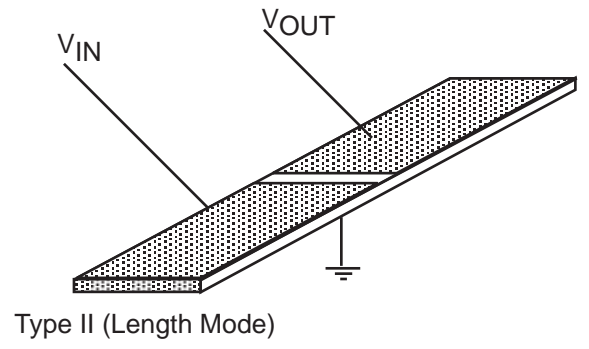
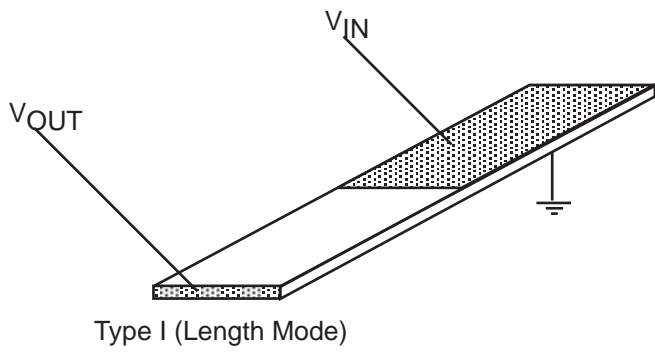


Fig 1. Several Piezoelectric Transformer Configurations

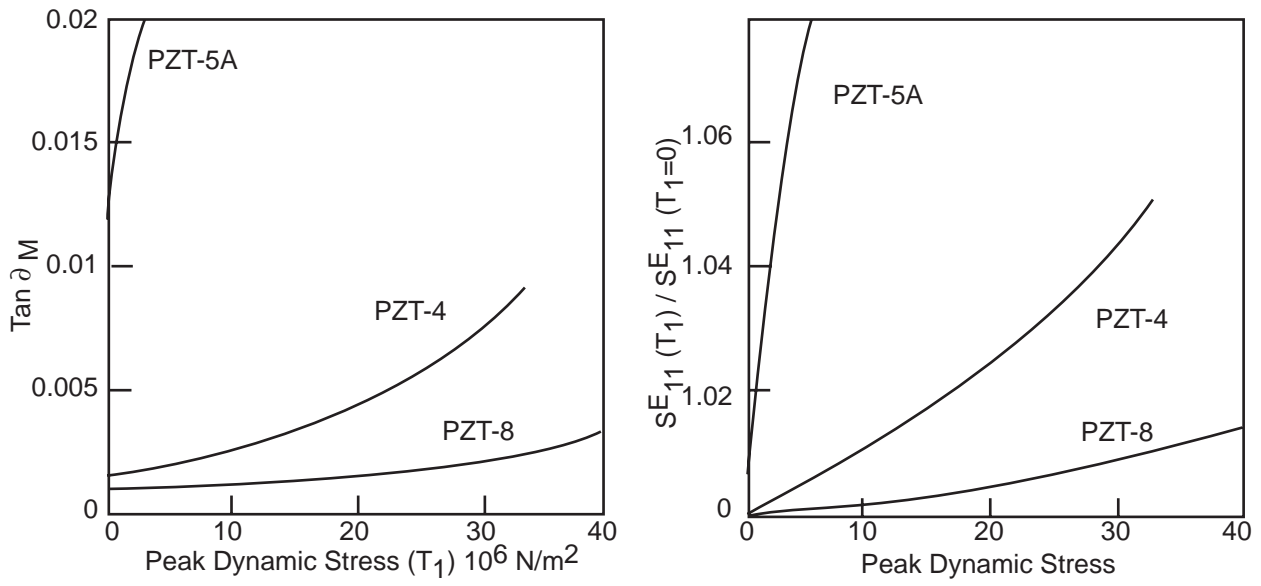


Fig 2a.  $\text{Tan } \delta_M$  and  $S_{E_{11}}^E$  vs Dynamic Stress  $(T_1)$  for several  $\text{Pb}(\text{Zr},\text{Ti})\text{O}_3$  Ceramics

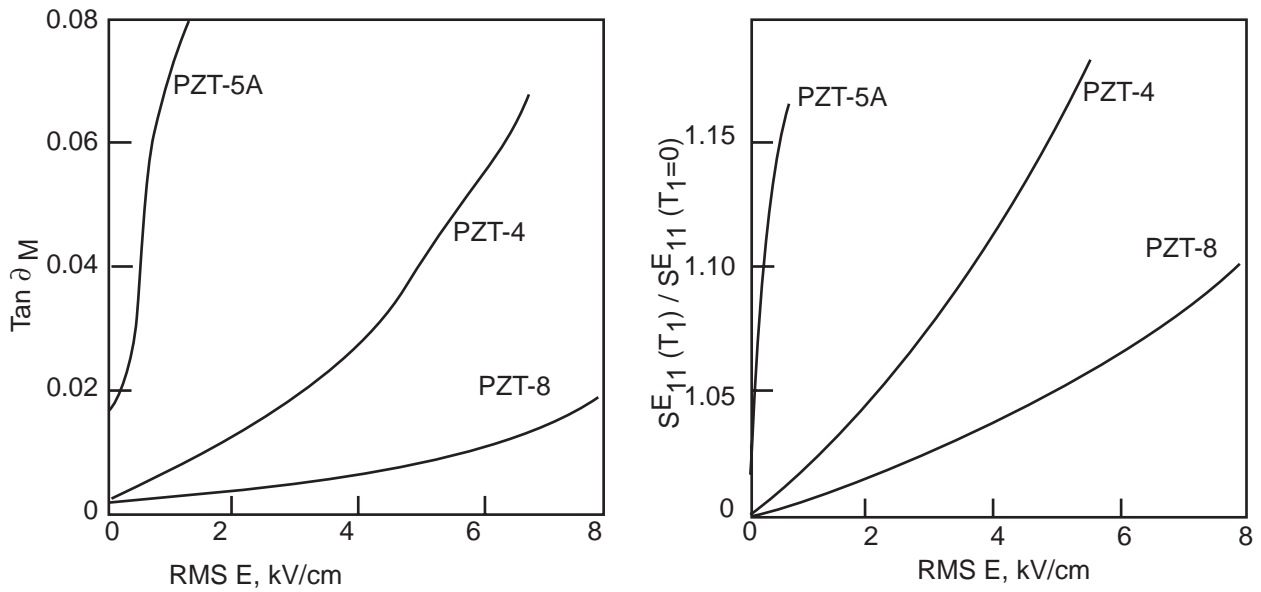
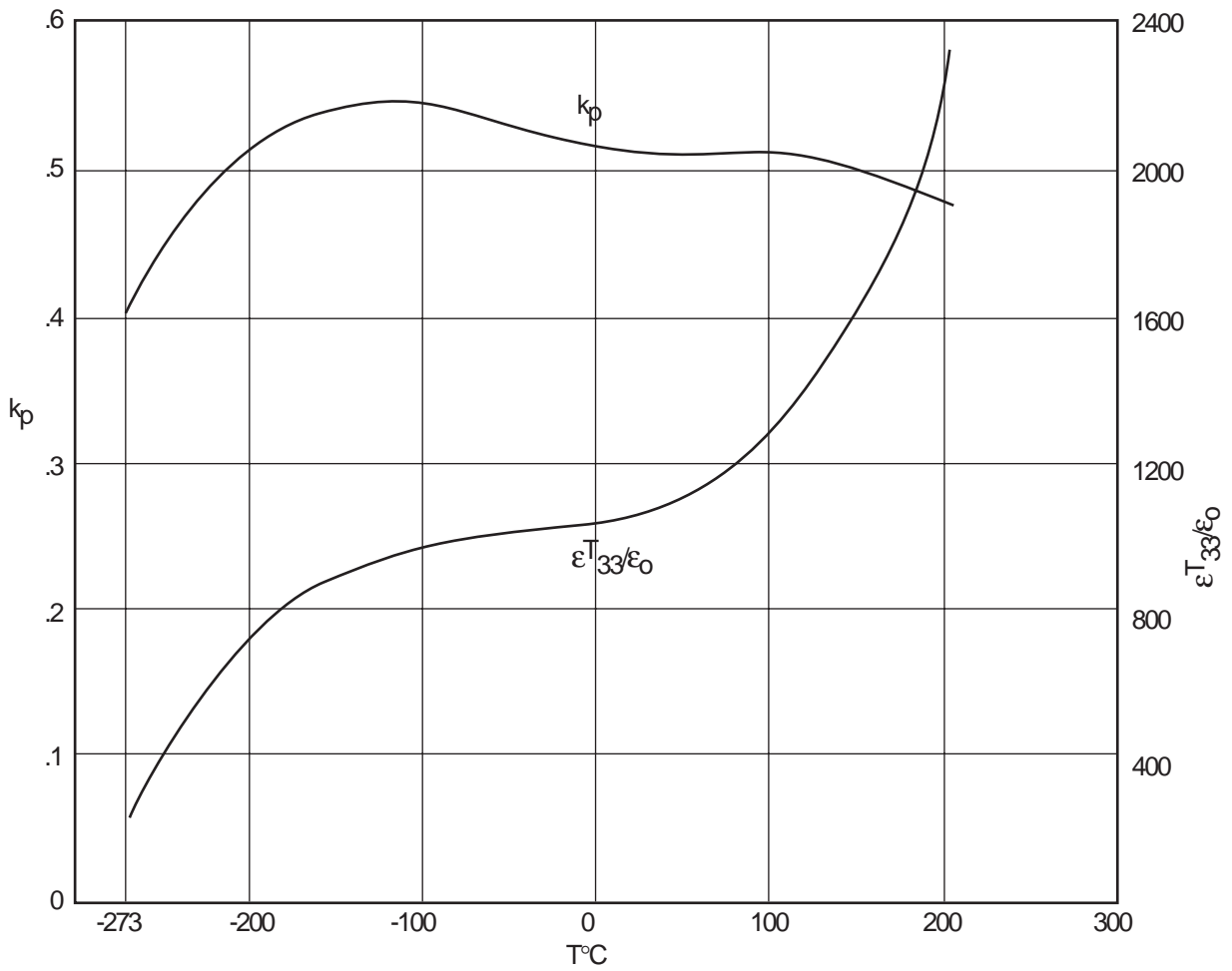
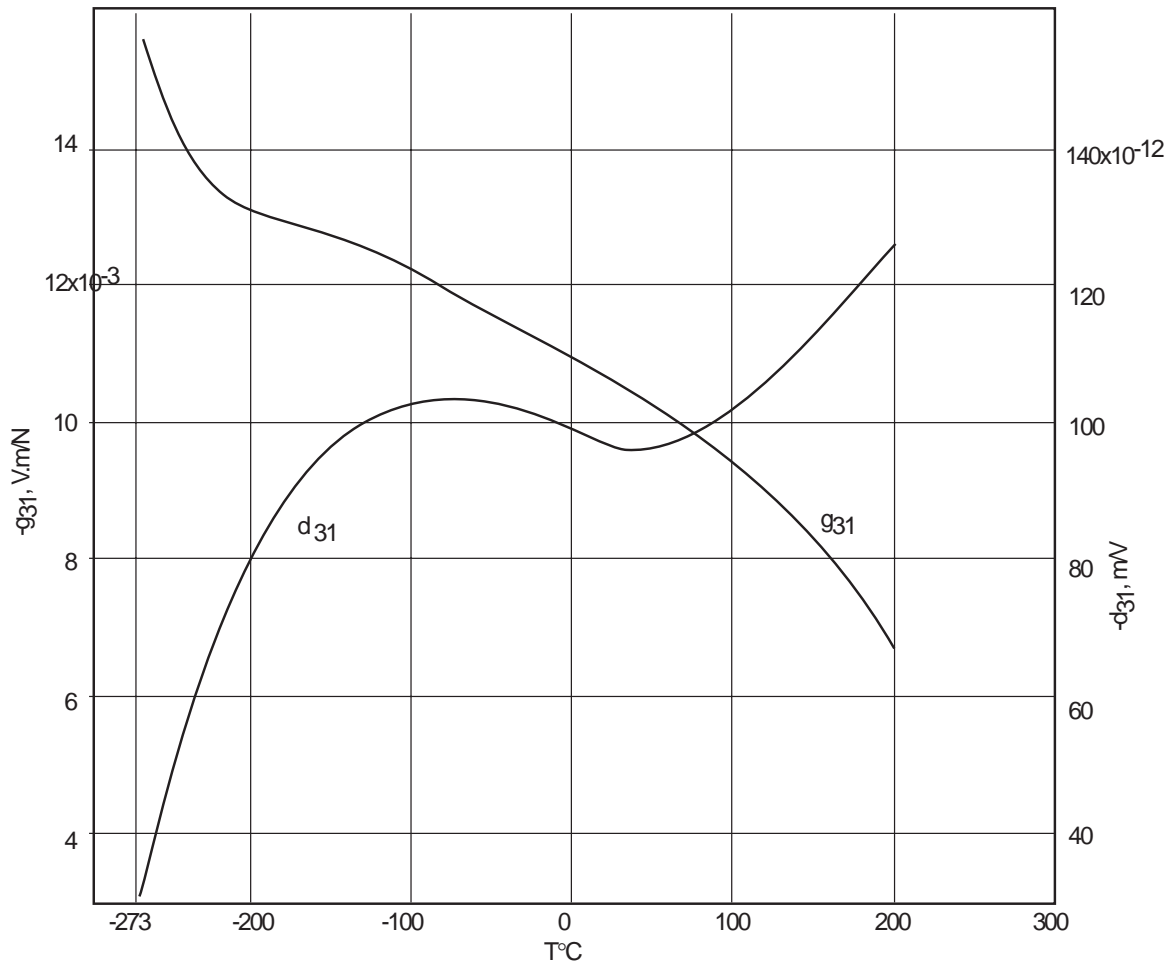


Fig 2b.  $\text{Tan } \delta_E$  and  $\epsilon_{33}^T$  vs Electric Field for several  $\text{Pb}(\text{Zr},\text{Ti})\text{O}_3$  Ceramics



$\epsilon^T_{33}/\epsilon_0$  and  $k_p$  vs Temp, PZT-8

Fig 3.  $\text{Tan}\delta_e$  and  $\epsilon^T_{33}/\epsilon_0$  and  $K_p$  vs Temperature. PZT-8



$d_{31}$  and  $g_{31}$  vs Temp, PZT-8

